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On the stochastic quantisation of Yang–Mills field theory

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Abstract. A new functional approach to the stochastic quantisation of continuum Euclidean Yang-Mills field theory is presented. Non-negative integral representations for the non-equilibrium and equilibrium probability distributions are given. A new dynamical generating function is proposed and its equilibrium limit is obtained. The new dynamical Feynman rules for the generating function are derived and the superficial renormalisability of the theory is studied. Euclidean volume divergences appear. The cancellations of the latter for any Euclidean dimension d, and, for $d = 4$, that of all quartic ultraviolet divergences, are carried out.

1. Introduction

The standard covariant quantisation of Yang-Mills (YM) gauge field theory in the path integral approach is based upon the introduction of a gauge fixing term which leads to the Faddeev-Popov ghosts [1] (see also, for example, [2]). However, for suitably large gauge fields, the gauge-fixing term is not sufficient either to fix the gauge uniquely or to ensure that the (integrand of the) resulting path integral be non-negative (at least for Euclidean fields). This ambiguity was suggested by Gribov [3] and, since then, analysed by several authors [4]. In 1981, Parisi and Wu [5] proposed an alternative procedure (called later stochastic quantisation) for quantising Euclidean field theories, including the non-Abelian YM gauge one. Originally, they introduced an artificial fifth time coordinate, t, in addition to the usual four Euclidean variables and assumed the system to evolve according to stochastic differential equations, i.e. Langevin equations with an external Gaussian white noise or, equivalently, Fokker-Planck equations for the probability density of field configurations at a given time t (for an introduction to stochastic differential equations, see, for example [6]). Since then, the subject of stochastic quantisation of Euclidean field theory has evolved in different directions [7]. We refer to [8] for an updated review. Further contributions to the stochastic quantisation of the continuum Euclidean non-Abelian YM field theory [9-12] will be summarised in § 2.

On the other hand, there is a wide knowledge of stochastic differential equations and related functional methods coming from critical dynamics in statistical physics [13-16].

In this paper, we shall present a new functional approach to stochastic quantisation of the continuum Euclidean YM theory which completely incorporates previous work in non-equilibrium statistical dynamics [14-16]. In § 3, we shall give interesting functional integrals for the non-equilibrium and equilibrium distributions.

Section 4 will present a dynamical generating function for the stochastically quantised YM field theory and some analysis of its equilibrium (or static) limit, and of correlation functions, by using the path integral discussed in § 3. Euclidean volume divergences will appear in the dynamical generating function. Recently there have been several contributions to the functional formulation of stochastic quantisation [17] but, to our knowledge, they are all limited to Abelian gauge theories.

In § 5, we shall use the dynamical generating function in order to derive the dynamical (or stochastic) Feynman rules. Through a power counting analysis, we shall study the superficial degree of divergence and renormalisability of the set of all dynamical Feynman diagrams in Euclidean dimension $d=4$, and point out the existence of quartic ultraviolet divergences in some diagrams. In § 6 we shall study the possibility of cancelling (i) the Euclidean volume divergences for any d, as well as (ii) the quartic ultraviolet divergences for $d=4$.

Finally, the conclusions are given.

2. Stochastic quantisation of the Euclidean Yang–Mills field: a summary

We shall consider the stochastic quantisation of the continuum Euclidean non-Abelian YM real gauge field $A^a_\mu = A^a_\mu(x,t)$ where the Euclidean vector $x=(x_\mu)$ has d components (including the usual time coordinate, which has now become another Euclidean component), $\mu(=1\ldots d)$ is a Lorentz index, a is a colour index and t is an artificial time parameter. According to [5, 9–12], stochastic quantisation of the YM field is based upon the Langevin equation

$$\partial A^a_\mu(x,t)/\partial t = -\gamma_0 \Lambda^a_{v\mu}(x,t) + y^a_\mu(x,t) \tag{2.1}$$

$$\Lambda^a_{v\mu}(x,t) = \frac{\delta S}{\delta A^a_\mu(x,t)} - D^{ab}_\mu V^b \qquad S = \int \mathrm{d}^d x\, \tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \tag{2.2}$$

$$\begin{aligned} F^a_{\mu\nu} &= F^a_{\mu\nu}(x,t) = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_0 f^{abc} A^b_\mu A^c_\nu \\ D^{ab}_\mu &= D^{ab}_\mu(x,t) = \partial_\mu \delta^{ab} - g_0 f^{abc} A^c_\mu \qquad \left(\partial_\mu \equiv \frac{\partial}{\partial x_\mu}\right) \end{aligned} \tag{2.3}$$

where S and D^{ab}_μ are the standard Euclidean YM action and covariant derivative, respectively, and $V^b = V^b(A,x)$ is an arbitrary gauge non-invariant (gauge-fixing) functional of x and A^a_μ. The constants f^{abc} and g_0 are the SU(N) structure constants and the gauge coupling constant, respectively. $y^a_\mu(x,t)$ is a Gaussian fluctuating force with zero average and such that the correlation of $y^a_\mu(x,t)$ and $y^b_\nu(x',t')$ equals $2\gamma_0 \delta^{ab}\delta_{\mu\nu}\delta^{(d)}(x-x')\delta(t-t')$, γ_0 (the diffusion coefficient) being the same constant which appears in equation (2.1). Standard summation conventions for repeated indices will be used throughout. Equivalently, the stochastic quantisation of the Euclidean YM theory can also be based upon the following Fokker–Planck equation for the probability distribution $P_V[A,t]$ [5, 9–12]:

$$\begin{aligned} L_V P_V[A,t] &\equiv \gamma_0 \int \mathrm{d}^d x \frac{\delta}{\delta A^a_\mu(x)} \left[\left(\frac{\delta}{\delta A^a_\mu(x)} + \Lambda^a_{V\mu}\right) P_V[A,t]\right] \\ &= \frac{\partial P_V[A,t]}{\partial t}. \end{aligned} \tag{2.4}$$

The relationship and equivalence between the Langevin and Fokker-Planck formulations in stochastic theory, with wide generality, are well documented (see, for example [6, 14].

The following properties turn out to play a fundamental role in the stochastic quantisation of the YM field and its subsequent physical interpretation.

(1) For any V such that $D_\mu^{ab} V^b \neq 0$, the solution $P_V[A, t]$ of equation (2.4) corresponding to an arbitrarily given initial condition $P_0[A]$ at $t = t_0$ relaxes as $t \to +\infty$ to a well defined equilibrium distribution $P_{V,\text{eq}}[A] = \lim_{t\to+\infty} P_V[A, t]$. Moreover, $P_{V,\text{eq}}[A]$ is (i) independent of $P_0[A]$, (ii) does depend on V, (iii) satisfies

$$L_V P_{V,\text{eq}}[A] = 0 \tag{2.5}$$

and (iv) for given V, it is unique. These properties were studied in [9-11].

(2) Let $F[A]$ be a generic gauge-invariant function of A_μ^a, i.e. $D_\mu^{ab}\delta F[A]/\delta A_\mu^b(x) = 0$. Since $A_\mu^a = A_\mu^a(xt)$ evolves with t according to the V-dependent Langevin equation (2.1), $F[A]$ also varies with t, but $\partial F/\partial t$ turns out to be independent of V [11]. The average value of such a gauge-invariant function is given by the functional integral

$$\langle F\rangle = \int [\mathrm{d}A] F[A] P_V[A, t] \tag{2.6}$$

where $[\mathrm{d}A] \equiv N \prod_{x,\mu,a} \mathrm{d}A_\mu^a(x)$ is the usual differential ('volume') element corresponding to the continuous set formed by all $A_\mu^a(x)$ for any x, μ, at a given (artificial) time t, while N is a suitable normalising factor. Then, as also discussed in [9-11], for any given initial condition $P_0[A]$ at $t = t_0$, (i) $(\partial/\partial t)\langle F\rangle$ is independent of V, in $t_0 < t < +\infty$ and (ii) $\lim_{t\to+\infty}\langle F\rangle$ exists, is independent of $P_0[A]$ and V and coincides with the average value of F given by by the Euclidean YM theory quantised by means of the well known Faddeev-Popov procedure, namely

$$\lim_{t\to+\infty}\langle F\rangle = \int [\mathrm{d}A] F[A] P_{V,\text{eq}}[A] = \int [\mathrm{d}A] F[A] P_{\text{FP}}[A]. \tag{2.7}$$

We recall that the Faddeev-Popov distribution is

$$P_{\text{FP}}[A] = \int [\mathrm{d}c\, \mathrm{d}\bar{c}] \exp[-(S + S_{\text{GF}})] \tag{2.8}$$

where S is given by equation (2.2), c and $\bar{c}$ are the usual ghost and antighost fields, $[\mathrm{d}c\,\mathrm{d}\bar{c}]$ is the standard 'volume' element for them (which may include a normalisation factor) and

$$S_{\text{GF}} = \int \mathrm{d}^d x\{(2\alpha_0)^{-1}(\partial_\mu A_\mu^a)(\partial_\nu A_\nu^a) + [\partial_\mu \bar{c}^a][D_\mu^{ab} c^b]\}. \tag{2.9}$$

3. Path integrals

By generalising previous work by other authors [6, 14], the solution $P_V[A, t]$ of the Fokker-Planck equation (2.4) in $t > t_0$ which is determined by the initial value $P_0[A]$ at $t = t_0$ can be cast as the functional integral

$$P_V[A, t] = \int [\mathrm{d}A'] Q_V[At; A't_0] P_0[A'] \tag{3.1}$$

where, in turn, the conditional probability distribution Q_V can be expressed through the path integral

$$Q_V[At; A't_0] = \int [DA] \exp\left\{ -\int_{t_0}^{t} dt' \int d^d x \left[\frac{1}{4\gamma_0} \left(\frac{\partial A^a_\mu}{\partial t'} + \gamma_0 \Lambda V^a_\mu \right) \times \left(\frac{\partial A^a_\mu}{\partial t'} + \gamma_0 \Lambda V^a_\mu \right) - \frac{1}{2} \frac{\delta}{\delta A^a_\mu(x, t')} \left(\frac{\partial A^a_\mu}{\partial t'} + \gamma_0 \Lambda V^a_n \right) \right] \right\} \tag{3.2}$$

$$[DA] = \prod [dA(t')] \qquad A^a_\mu(x, t_0) = A'^a_\mu(x) \qquad A^a_\mu(x, t) = A^a_\mu(x). \tag{3.3}$$

Notice that $[dA(t')]$ is a differential element at a given time t' which is similar to the one appearing in equations (2.6) and (2.7).

Property (1), summarised in § 2, ensures that, after an infinitely long time interval has elapsed, all memory effects become lost and the solution of the Fokker-Planck equation relaxes to the equilibrium distribution, $P_{V,\mathrm{eq}}$. This fact, together with equation (3.2) gives the path integral representation for $P_{V,\mathrm{eq}}[A]$:

$$P_{V,\mathrm{eq}}[A] = \int [DA] \exp\left\{ -\int_{-\infty}^{t} dt' \int d^d x \left[\frac{1}{4\gamma_0} \left(\frac{\partial A^a_\mu}{\partial t'} + \gamma_0 \Lambda V^a_\mu \right) \times \left(\frac{\partial A^a_\mu}{\partial t'} + \gamma_0 \Lambda V^a_\mu \right) - \frac{1}{2} \frac{\delta}{\delta A^a_\mu(x, t')} \left(\frac{\partial A^a_\mu}{\partial t'} + \gamma_0 \Lambda V^a_\mu \right) \right] \right\} \tag{3.4}$$

with $A^a_\mu(x, t) = A^a_\mu(x)$ for *any* fixed t and *any initial* configuration $A^a_\mu(x, t' \to -\infty) = A'^a(x)$ at $t_0 = t' \to -\infty$.

Notice that for real V, equation (3.4) gives a representation for the equilibrium distribution of the Euclidean YM field which is non-negative for an A and, hence, it provides a whole family of interesting alternatives with respect to the Faddeev-Popov distribution, which, in particular, could be useful for a non-perturbative analysis. See also the discussion in [9, 10, 12]. For fixed t_0 and $t \to +\infty$, equation (3.2) yields, through a similar argument

$$\int [DA] \exp\left\{ -\int_{t_0}^{+\infty} dt' \int d^d x \left[\frac{1}{4\gamma_0} \left(\frac{\partial A^a_\mu}{\partial t'} + \gamma_0 \Lambda V^a_\mu \right) \left(\frac{\partial A^a_\mu}{\partial t'} + \gamma_0 \Lambda V^a_\mu \right) - \frac{1}{2} \frac{\delta}{\delta A^a_\mu(xt')} \left(\frac{\partial A^a_\mu}{\partial t'} + \gamma_0 \Lambda V^a_\mu \right) \right] \right\} = P_{V,\mathrm{eq}}[A] \equiv N_\infty \tag{3.5}$$

for any $A^a_\mu(x, t_0) = A'^a_\mu(x)$ at any fixed t_0. *The final* field configuration is $A^a_\mu(x, t' \to +\infty) = A^a_\mu(x)$.

4. Dynamical generating functions

We recall that the equilibrium generating function and the normalisation constant N_{eq} are defined as

$$Z_{V,\mathrm{eq}}[j] = N_{\mathrm{eq}} \int [dA] \exp\left(\int d^d x\, j^a_\mu(x) A^a_\mu(x) \right) P_{V,\mathrm{eq}}[A] \tag{4.1}$$

where $j = j^a_\mu(x)$ is a t-independent external current. A study of $Z_{V,\mathrm{eq}}[j]$ and of its perturbation expansion when $V^a = \alpha_0^{-1} \partial_\mu A^a_\mu$, based upon equation (2.5), was given by Floratos *et al* [12].

We shall introduce the following dynamical generating function related to the Fokker-Planck equation (2.4):

$$Z_V[J]=N\int[DA]\exp\left\{\int_{-\infty}^{+\infty}\mathrm{d}t\int\mathrm{d}^dx\left[J^a_\mu A^a_\mu-\frac{1}{4\gamma_0}\left(\frac{\partial A^a_\mu}{\partial t}+\gamma_0\Lambda^a_{V\mu}\right)\right.\right.$$
$$\left.\left.\times\left(\frac{\partial A^a_\mu}{\partial t}+\gamma_0\Lambda^a_{V\mu}\right)+\frac{1}{2}\frac{\delta}{\delta A^a_\mu(x,t)}\left(\frac{\partial A^a_\mu}{\partial t}+\gamma_0\Lambda^a_{V\mu}\right)\right]\right\} \tag{4.2}$$

where $J=J^a_\mu(x,t)$ is a t-dependent external current, $[DA]=\Pi^{+\infty}_{t=-\infty}[\mathrm{d}A(t)]$ and the normalisation constant N fulfils

$$Z_V[J=0]=1. \tag{4.3}$$

The term

$$\exp\int_{-\infty}^{+\infty}\mathrm{d}t\int\mathrm{d}^dx\frac{1}{2}\frac{\delta}{\delta A^a_\mu(x,t)}\left[\frac{\partial A^a_\mu}{\partial t}\right]$$

on the right-hand side of equation (4.2) does not contribute to the path integral and can be safely absorbed into the normalisation constant.

This dynamical generating function turns out to be the direct generalisation for the actual YM field case of the one considered in [15] for the critical dynamics of mode-coupling systems (helium, antiferromagnets, liquid-gas systems). Let us consider equation (1.8) in [15] and perform the Gaussian functional integration over the auxiliary Martin-Siggia-Rose [18] field $\hat{\phi}_j$ conjugate to ϕ_j. Then the direct generalisation of the resulting functional integral for the generating functional Z $\{l\}$ in [15] to the YM case is just equation (4.2). We could introduce auxiliary Martin-Siggia-Rose gauge fields $\hat{A}^a_\mu$ conjugate to A^a_μ here as well. In short, their use would allow us, in a perturbation theory framework, to treat vertices having a relatively simple structure at the expense of having to deal with relatively complicated propagators related to $\hat{A}^a_\mu$ and A^a_μ. After some calculational attempts, we have preferred not to consider auxiliary fields for the time being but to work exclusively with the original real gauge field A^a_μ throughout this work, since, at the price of somewhat complicated vertices, we can restrict ourselves to treat just one kind of propagator and vertex function, namely those for the (unique) field A^a_μ.

Let

$$J^a_\mu(x,t)=j(x)\delta(t) \qquad \text{or} \qquad J=j\delta \tag{4.4}$$

symbolically, where J and j are the external currents which appear in equations (4.2) and (4.1). Then one has the following property which relates the dynamical generating functional Z_V to the equilibrium one:

$$Z_V[J=j\delta]=Z_{V,\mathrm{eq}}[j]. \tag{4.5}$$

Equation (4.5) can be proved as follows. By assuming equation (4.4) and recalling (3.2), equation (4.2) becomes

$$Z_V[J=j\delta]=N\int[\mathrm{d}A(0)]\exp\left(\int\mathrm{d}^dx\,j^a_\mu(x)A^a_\mu(x,0)\right)Q_V[A'',t=+\infty;A(0),t_0=0]$$
$$\times Q_V[A(0),t=0;A',t'_0=-\infty] \tag{4.6}$$

where $A'^a_\mu(x) = A'^a_\mu(x, t'_0 \to -\infty)$ and $A''^a_\mu(x) = A''^a_\mu(x, t' \to +\infty)$. By recalling equations (3.4) and (3.5) with $N_\infty = P_{V,\text{eq}}[A'']$, one finds

$$Z_V[J = j\delta] = NN_\infty \int [\mathrm{d}A(0)] \exp\left(\int \mathrm{d}^d x\, j^a_\mu(x) A^a_\mu(x)\right) P_{V,\text{eq}}[A]. \tag{4.7}$$

By performing the change of notation $A^a_\mu(x, 0) = A^a_\mu(x)$ and noticing that the resulting normalisation constant NN'_∞ is such that the right-hand side of equation (4.7) equals unity when $j = 0$, its left-hand side does as well by virtue of equation (4.3), one completes the proof of equation (4.5).

The 'two-point' correlation function for the gauge field is by generalising properties which are well known in stochastic theory (for instance, compare with sections 6.3–6.6 in [6]),

$$\langle A^a_\mu(x_1, t_1) A^b_\nu(x_2, t_2)\rangle = \int [\mathrm{d}A][\mathrm{d}A'] A^a_\mu(x_1) A'^b_\nu(x_2)\{\theta(t_1 - t_2)$$

$$\times Q_V[A, t_1; A', t_2] P_{V,\text{eq}}[A'] + \theta(t_2 - t_1) Q_V[A', t_2; A, t_1] P_{V,\text{eq}}[A]\} \tag{4.8}$$

θ being the usual step function: $\theta(t) = 1(0)$ for $t > 0 (t < 0)$. One has the representations

$$\langle A^a_\mu(x_1, t_1) A^b_\nu(x_2, t_2)\rangle = \left[\frac{\delta^2 Z_V[J]}{\delta J^a_\mu(x_1, t_1)\,\delta J^b_\nu(x_2, t_2)}\right]_{J=0} \tag{4.9}$$

In fact, by differentiating equation (4.2) functionally, one gets ($t_1 > t_2$, for definiteness)

$$\left[\frac{\delta^2 Z_V[J]}{\delta J^a_\mu(x_1, t_1)\,\delta J^b_\nu(x_2, t_2)}\right]_{J=0} = N \int [\mathrm{d}A(t_1)][\mathrm{d}A'(t_2)] A^a_\mu(x_1, t_1) A'^b_\nu(x_2, t_2)$$

$$\times Q_V[A'', t = +\infty; A(t_1), t_1] Q_V[A(t_1), t_1; A'(t_2), t_2]$$

$$\times Q_V[A'(t_2), t_2; A''', t' = -\infty] \tag{4.10}$$

where A'' and A''' are the gauge field configurations at $t \to +\infty$ and $t \to -\infty$, respectively. Equation (4.10) leads to (4.9), by recalling equations (3.4) and (3.5) through arguments similar to those used in connection with equations (4.6) in order to establish equation (4.5). In particular, for $t_1 = t_2 = t$, the equal-time or equilibrium correlation function fulfils, by virtue of equations (4.5) and (4.9),

$$\langle A^a_\mu(x_1, t) A^b_\nu(x_2, t)\rangle = \left[\frac{\delta^2 Z_V}{\delta J^a_\mu(x_1, t)\,\delta J^b_\nu(x_2, t)}\right]_{J=0} = \int [\mathrm{d}A] A^a_\mu(x_1) A^b_\nu(x_2) \tag{4.11}$$

$$P_{V,\text{eq}}[A] = \left[\frac{\delta^2 Z_{V,\text{eq}}}{\delta j^a_\mu(x_1)\,\delta j^b_\nu(x_2)}\right]_{j=0}.$$

5. Feynman rules from the dynamical generating functions

5.1. Free dynamical generating function

In what follows, we shall assume the same gauge-fixing functional as in [9] and [12], namely

$$V^a = \partial_\mu A^a_\mu / \alpha_0. \tag{5.1}$$

We shall introduce the free dynamical generating function

$$Z_{V,0}[J]=N^{(0)}\int [DA]\exp\left\{\int_{-\infty}^{+\infty}\mathrm{d}t\int \mathrm{d}^dx\left(J^a_\mu A^a_\mu-\frac{1}{4\gamma_0}\frac{\partial A^a_\mu}{\partial t}\frac{\partial A^a_\mu}{\partial t}-\frac{\gamma_0}{4}\Lambda^a_{V,0\mu}\Lambda^a_{V,0\mu}\right)\right\} \tag{5.2}$$

$$\Lambda^a_{V,0\mu}=-\Delta A^a_\mu+(1-1/\alpha_0)\partial_\mu(\partial_\nu A^a_\nu)\qquad \Delta A^a_\mu=(\partial^2/\partial x_\nu\,\partial x_\nu)A^a_\mu \tag{5.3}$$

the normalisation constant $N^{(0)}$ being such that $Z_{V,0}[0]=1$. $Z_{V,0}[J]$ can be obtained from Z_V, $[J]$ by (i) replacing $\Lambda^a_{V\mu}$ by $-\Delta A^a_\mu+\partial_\mu(\partial_\nu A^a_\nu)$, (ii) omitting the term $\frac{1}{2}\gamma_0$, $\delta\Lambda^a_{V\mu}/\delta A^a_\mu$ and (iii) accepting that the remaining contribution in the exponential inside the path integral, namely

$$-\frac{1}{2\gamma_0}\int_{-\infty}^{+\infty}\mathrm{d}t'\int \mathrm{d}^dx\frac{\partial A^a_\mu}{\partial t'}\gamma_0\left[-\Delta A^a_\mu+\left(1-\frac{1}{\alpha_0}\right)\partial_\mu(\partial_\nu \dot{A}^a_\nu)\right]$$

$$=-[S_{V,0}(t'=+\infty)-S_{V,0}(t'=-\infty)]$$

can be dropped, where $(A^a_\mu=A^a_\mu(xt'))$

$$S_{V,0}(t')=\int \mathrm{d}^dx\left[\frac{1}{4}(\partial_\mu A^a_\nu-\partial_n A^a_\mu)(\partial_\mu A^a_\nu-\partial_\nu A^a_\mu)+(1/2\alpha_0)(\partial_\mu A^a_\mu)(\partial_\nu A^a_\nu)\right].$$

By performing the Gaussian integration over A^a_μ through standard techniques [2], equation (5.2) becomes, after some algebra,

$$Z_{V,0}[J]=\exp\left(\tfrac{1}{2}\int_{-\infty}^{+\infty}\mathrm{d}t_1\int_{-\infty}^{+\infty}\mathrm{d}t_2\int \mathrm{d}^dx_1\,\mathrm{d}^dx_2 J^a_\mu(x_1,t_1)G^{ab}_{\mu\nu}(x_1-x_2,t_1-t_2)J^b_\nu(x_2,t_2)\right) \tag{5.4}$$

$$\tilde{G}^{ab}_{\mu\nu}(k,\omega)=\int_{-\infty}^{+\infty}\mathrm{d}t\int \mathrm{d}^dx\,G^{ab}_{\mu\nu}(x,t)\exp[-\mathrm{i}(kx-\omega t)]$$

$$=2\gamma_0\delta^{ab}\frac{[\omega^2+\gamma_0^2\alpha_0^{-2}(k^2)^2]\delta_{\mu\nu}+\gamma_0^2(1-\alpha_0^{-2})k^2k_\mu k_\nu}{[\omega^2+\gamma_0^2(k^2)^2][\omega^2+\gamma_0^2\alpha_0^{-2}(k^2)^2]}. \tag{5.5}$$

As a consistency check of our formalism and, in particular, of equation (4.5) we consider equation (5.4) for the special external current given in (4.4). Then (5.4) becomes

$$Z_{V,0}[J=j\delta]=\exp\left(\tfrac{1}{2}\int \mathrm{d}^dx_1\,\mathrm{d}^dx_2\,j^a_\mu(x_1)G^{ab}_{\mathrm{eq}\mu\nu}(x_1-x_2)j^b_\nu(x_2)\right) \tag{5.6}$$

$$\tilde{G}^{ab}_{\mathrm{eq}\mu\nu}(k)=\int \mathrm{d}^dx\,G^{ab}_{\mathrm{eq}\mu\nu}(x)\exp(-\mathrm{i}kx)=\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega}{2\pi}\tilde{G}^{ab}_{\mu\nu}(k)=\frac{\delta^{ab}}{k^2}\left(\delta_{\mu\nu}+(\alpha_0-1)\frac{k_\mu k_\nu}{k^2}\right) \tag{5.7}$$

where equation (5.5) has been used and a residue integration has been performed. The right-hand side of equation (5.6) is the free (equilibrium) generating function for the Abelian ($g_0=0$) and Euclidean gauge field with Euclidean propagator given by equation (5.7): compare with [2].

A posteriori, one also realises the interest (the necessity!) of having introduced a non-vanishing gauge-fixing function V in equations (2.1), (2.2) and (2.4). In fact, by letting $\alpha_0 > \infty$ in the specific choice (5.1), one would be faced with a divergent result in equation (5.7)!

5.2. *Feynman rules*

The perturbation expansions of the dynamical generating functions in equations (4.2) and (4.3) are obtained as power series in g_0 through standard techniques [2] by using the formal relation

$$Z_V[J] = \exp\left(-\int_{-\infty}^{+\infty} \mathrm{d}t \int \mathrm{d}^d x\, \mathscr{L}_{V,\mathrm{int}}\left[A_\mu^a(x,t) \to \frac{\delta}{\delta J_\mu^a(x,t)}\right]\right) Z_{V,0}[J] \tag{5.8}$$

$$\mathscr{L}_{V,\mathrm{int}} = \gamma_0\left[\tfrac{1}{4}\Lambda_{V\mu}^a \Lambda_{V\mu}^a - \frac{1}{2}\frac{\delta\Lambda_{V\mu}^a}{\delta A_\mu^a}\right] - \frac{\gamma_0}{4}\Lambda_{V0\mu}^a\Lambda_{V0\mu}^a + \frac{1}{2}\frac{\partial A_\mu^a}{\partial t}[\Lambda_{V\mu}^a - \Lambda_{V0\mu}^a]. \tag{5.9}$$

A careful study of the perturbative series generated by equations (5.8), (5.9), (5.4) and (4.10) for correlation functions show that, up to and including order g_0^2, the Feynman rules in (k, ω)-space reduce to considering just one line corresponding to the propagator in equation (5.5) and the vertices drawn in figure 1 for an arbitrary gauge parameter α_0. The rules are as follows.

(i) If an arrow of a line in figure 1 is to be reversed, the signs of the frequency and momentum associated with that line are to be changed.

(ii) As can be seen in figure 1, the contribution associated with vertex I depends on just two of the momenta of the four external legs. The crosses over the lines attached to the vertices in figure 1 indicate that a crossed leg with momentum k contributes with a factor like $[(1-1/\alpha_0)k_\nu\delta_{\lambda\mu} - 2k_\mu\delta_{\lambda\nu} + k_\lambda\delta_{\mu\nu}]$, ν, μ being the Lorentz indices of the crossed line and of one of the other uncrossed lines, respectively, and λ is a dummy summation index.

(iii) The small circle over one of the lines in vertices II and III in figure 1 indicate that the contributions associated with those vertices depend precisely on the frequency ω and momentum k of the circled line through a factor $[(-\mathrm{i}\omega + \gamma_0 k^2)\delta_{\mu\rho} - \gamma_0 k_\mu k_\rho(1-1/\alpha_0)]$ where μ is the Lorentz index of the circled line and ρ is a dummy summation index.

(iv) Include a global factor $1/n!$ where n is the number of vertices.

(v) The contribution associated with any vertex in figure 1 is to be multiplied by $(2\pi)^{d+1}\,\delta(\Sigma\text{ (frequencies)})\,\delta^{(d)}(\Sigma\text{ (wavevectors)})$. Here, Σ (frequencies) (Σ (wavevectors)) stand for the sum of all frequencies (wavevectors) corresponding to all lines ending at the vertex.

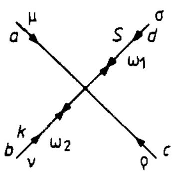
(vi) As usual, internal lines with frequency ω and wavevector k are to be integrated with $\int_{-\infty}^{+\infty}(\mathrm{d}\omega/2\pi)\int(\mathrm{d}^d k/(2\pi)^d$. All these lead to extract a factor $(2\pi)^{-d-1}$ times a $(d+1)$-dimensional δ function expressing the total frequency-momentum conservation.

(vii) The vertices in figure 1 containing the 'Euclidean volume' divergence $\delta^{(d)}(0)$ comes precisely from the term $(\gamma_0/2)\delta\Lambda_{V\mu}^a/\delta A_\mu^a$ in both equations (5.9) and (5.10). Notice that $\delta_{\mu\mu} = d$.

(viii) No use of symmetry factory is needed here as we have not symmetrised the vertices. It is, then, necessary to carefully count the number of equivalent ways of drawing a particular graph. The complexity of dealing with crosses and bubbles and counting equivalent diagrams disappears when the vertices are symmetrised. However, it is easier to show with the present Feynman rules the cancellation among the Euclidean

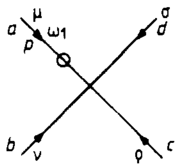
Vertex — Associated contribution

I



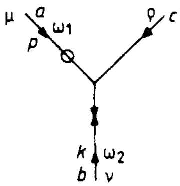
$f^{eab}f^{ecd}g_0^2\tfrac{1}{4}\gamma_0[(1-1/\alpha_0)k_\nu\delta_{\lambda\mu}-2k_\mu d_{\lambda\nu}+k_\lambda\delta_{\mu\nu}][(1-1/\alpha_0)S_\sigma\delta_{\lambda\rho}-2S_\rho\delta_{\lambda\sigma}+S_\lambda\delta_{\rho\sigma}]$

II



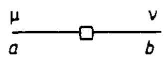
$f^{eab}f^{ecd}\tfrac{1}{2}g_0^2[(-\mathrm{i}\omega_1+\gamma_0p^2)\delta_{\mu\rho}-\gamma_0p_\mu p_\rho(1-1/\alpha_0)]\delta_{\nu\sigma}$

III



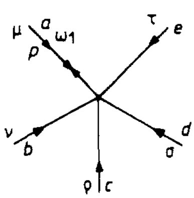
$\mathrm{i}f^{abc}\tfrac{1}{2}g_0[(-\mathrm{i}\omega_1+\gamma_0p^2)\delta_{\mu\sigma}-\gamma_0p_\mu p_\sigma(1-1/\alpha_0)][(1-1/\alpha_0)k_\nu\delta_{\sigma\rho}-2k_\rho\delta_{\sigma\nu}+k_\sigma\delta_{\rho\nu}]$

IV



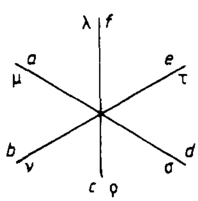
$\tfrac{1}{2}\gamma_0g_0^2f^{eda}f^{edb}\delta^{(d)}_{(0)}(\delta_{\lambda\lambda}-1)\delta_{\mu\nu}$

V



$\mathrm{i}\gamma_0\tfrac{1}{2}g_0^3f^{a'ab}f^{a'b'c}f^{b'ed}[(1-1/\alpha_0)p_\nu\delta_{\lambda\mu}-2p_\mu\delta_{\lambda\nu}+p_\lambda\delta_{\mu\nu}]\delta_{\rho\sigma}\delta_{\lambda\tau}$

VI



$-\tfrac{1}{4}g_0^4f^{a'b'a}f^{a'bc}f^{c'b'd}f^{c'ef}\delta_{\mu\rho}\delta_{\gamma\tau}\delta_{\lambda\sigma}$

Figure 1. Feynman rules for $Z_{V,L}$.

volume divergence of vertex IV with other graphs coming from the g_0^2 correction to the gauge field propagator. The symmetrisation here avoided will be analysed in a forthcoming paper where we will calculate the whole gauge-field propagator up to order g_0^2.

Note that these Feynman rules are different from those considered by Alfaro, for instance, in [7], (see also [8]). In their case, the fictitious time was integrated between

0 and a finite value, while in our approach it is integrated between $-\infty$ and $+\infty$, allowing us to go over to Fourier space in the artificial time also.

5.3. Power counting and superficial degree of divergence

We use the Feynman rules associated with all vertices corresponding to Z_V. We consider a general graph with n_i vertices of type i (= I, II, III, V and VI), L integrals over pairs (p, ω), i.e. over independent interval momenta and frequencies, I internal lines and E external legs. For each integration over an internal momentum a factor Λ^d is obtained, Λ being a momentum ultraviolet cut-off so that we end up with Λ^{Ld}. The integrals over frequencies produce a Λ^{-2L} factor. The remaining $I-L$ propagators yield a factor $\Lambda^{4(L-I)}$. On the other hand, the n_i vertices give at most an overall factor Λ^{2n_i} $(i = \text{I, II})$, Λ^{3n_i} $(i = \text{III})$. Λ^{n_i} $(i = \text{V})$, and the vertex VI is Λ independent, as can be seen in figure 1. So, finally, the degree of divergence δ is *at most* $\delta = (\frac{1}{2}n - \frac{1}{2}E + 1)d - 2n + E + 2$, where $n = 2n_1 + 2n_2 + n_3 + 3n_5 + 4n_6$ is the order in perturbation theory of the considered graph. We have not considered vertices corresponding to $i = \text{IV}$, containing 'Euclidean volume' divergences because they can be shown to disappear for any d (not only for $d = 4$) through an interesting cancellation mechanism which involves other Feynman diagrams for Z_V, as will be seen in § 6.

It may be easily seen that in $d = 4$ the degree of divergence δ is independent of the order n of the graph considered, so that the theory is naively or superficially renormalisable in $d = 4$. The previously obtained formula for δ should be supplemented with two additional and interesting results. First, let us suppose that $\delta = 4$ does occur, in fact, for a particular graph (as, will be the case for graphs (a), (b) and (c) in figure 2. It turns out that this divergence degree is reduced to, at least, $\delta = 2$, either by

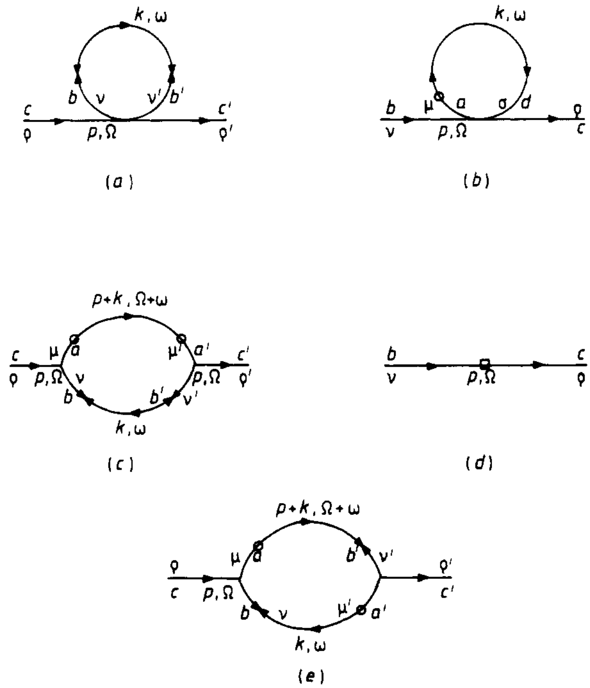


Figure 2. Quartically divergent graphs.

cancellations with other graphs or by appealling to dimensional regularisation, as will be seen in § 6. Second, and as we will show in a forthcoming paper, this quadratic divergence ($\delta = 2$) is reduced to a logarithmic one by means of, for instance, dimensional regularisation techniques [19].

6. Cancellation of quartic ultraviolet divergences

For $d = 4$, the vertices represented in figure 1 give rise, as has been announced above, to quartic ultraviolet divergences in general.

The graphs which, up to order g_0^2, are quartically divergent are represented in figure 2. We will compute the amputated graphs, i.e. without their external legs. Notice that the 'Euclidean volume' divergence of order g_0^2 has to be included, since it can also be viewed as a quartic ultraviolet divergence for $d = 4$. As we are going to see, the quartic divergence due to graph (a) cancels exactly the one arising from (c) while the other two (i.e. (b) and (d)) also cancel with each other. We stress that the cancellations to be presented below are not restricted to $d = 4$ but are, in fact, valid for any d.

By using the Feynman rules for Z_V given in figure 1, it is straightforward to obtain

$$(a) = 2\left(-g_0^2\frac{\gamma_0}{4}\right)f^{acb}f^{ac'b'}\int\frac{\mathrm{d}^dk\,\mathrm{d}\omega}{(2\pi)^{d+1}}\tilde{G}^{bb'}_{\nu\nu'}(k,\omega)\left[\left(1-\frac{1}{\alpha_0}\right)k_\nu\delta_{\sigma\rho}\right.$$
$$\left.-2k_\rho\delta_{\nu\sigma}+k_\sigma\delta_{\rho\nu}\right]\left[\left(1-\frac{1}{\alpha_0}\right)k_{\nu'}\delta_{\sigma\rho'}-2k_{\rho'}\delta_{\sigma\nu'}+k_\sigma\delta_{\rho'\nu'}\right]. \tag{6.1}$$

The factor 2 comes from the permutation of the external legs, i.e. the interchange of the indices $(\rho, c)\leftrightarrow(\rho', c')$ which gives rise to a similar graph.

We now need to compute the graph (c) in order to see the cancellation. Notice that, *a priori*, the Feynman rules for Z_V look a bit complicated since they involve frequency dependences at vertices of types II and III of figure 1. Nevertheless, it turns out that these rules allow for a simpler calculation of graph (c) in figure 2, since in that case the denominators can be reduced due to the following remarkable formula:

$$\left[\left(\omega^2+\gamma_0^2\frac{(k^2)^2}{\alpha_0^2}\right)\delta_{\mu\mu'}+\gamma_0^2k^2k_\mu k_{\mu'}\left(1-\frac{1}{\alpha_0^2}\right)\right]\left[(-\mathrm{i}\omega+\gamma_0k^2)\delta_{\mu\sigma}-\gamma_0k_\mu k_\sigma\left(1-\frac{1}{\alpha_0}\right)\right]$$
$$\times\left[(\mathrm{i}\omega+\gamma_0k^2)\delta_{\mu'\sigma'}-\gamma_0k_{\mu'}k_{\sigma'}\left(1-\frac{1}{\alpha_0}\right)\right]$$
$$=\left(\omega^2+\gamma_0^2\frac{(k^2)^2}{\alpha_0^2}\right)(\omega^2+\gamma_0^2(k^2)^2)\delta_{\sigma\sigma'}. \tag{6.2}$$

By using the Feynman rules for Z_V and the last formula (6.2) we obtain

$$(c) = 2\frac{1}{2!}\frac{g_0^2}{4}f^{abc}f^{ab'c'}\int\frac{\mathrm{d}^dk\,\mathrm{d}\omega}{(2\pi)^{d+1}}\tilde{G}^{bb'}_{\nu\nu'}(k,\omega)\left[\left(1-\frac{1}{\alpha_0}\right)k_\nu\delta_{\sigma\rho}\right.$$
$$\left.-2k_\rho\delta_{\sigma\nu}+k_\sigma\delta_{\rho\nu}\right]\left[\left(1-\frac{1}{\alpha_0}\right)k_{\nu'}\delta_{\sigma\rho'}-2k_{\rho'}\delta_{\sigma\nu'}+k_\sigma\delta_{\rho'\nu'}\right] \tag{6.3}$$

where the factor 2 comes from the permutation of the external lines, i.e. of the indices $(\rho, c)\rightarrow(\rho', c')$. It is now easy to see that the sum of graphs (c) (equation (6.3)) and

(a) (equation (6.1)) gives rise to a vanishing result. In order to complete the cancellation we need the well known result

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} F(k^2) k_\mu k_\nu = \frac{\delta_{\mu\nu}}{d} \int \frac{\mathrm{d}^d k}{(2\pi)^d} F(k^2) \tag{6.4}$$

where $F(k^2)$ is an arbitrary function of k^2.

There are four different ways in which graph (b) may be constructed: two of them are equal to each other, and the other two, which are also equal, are the complex conjugates of the first ones. Using this and (6.4) we find

$$(b) = -\gamma_0 g_0^2 f^{eab} f^{eac} \int \frac{\mathrm{d}^d k}{(2\pi)^d} (\delta_{\mu\mu} - 1)\delta_{\nu\rho}. \tag{6.5}$$

By computing the graph (d) now the following result is obtained which clearly cancels the last one (6.5):

$$(d) = 2\tfrac{1}{2}\gamma_0 g_0^2 f^{edb} f^{edc} \delta^{(d)}(0)(\delta_{\mu\mu} - 1)\delta_{\nu\rho}. \tag{6.6}$$

The last type of cancellation between (b) and (d) is the same as that we found in a previous work on the dynamics of the Ginzburg–Landau theory [16] (see also [13, 15]). It is characterised by the fact that it is α_0 independent (gauge independent). On the other hand, the first cancellation between (a) and (c) seems to be an intrinsic feature of gauge theory as the gauge parameter α_0 is involved in it. Finally, graph (e) in figure 2 can be seen to vanish. It is straightforward to obtain from (6.2)

$$\begin{aligned}
&\left[\left(\omega^2 + \gamma_0^2 \frac{(k^2)^2}{\alpha_0^2}\right)\delta_{\mu\nu'} + \gamma_0^2 k^2 k_\mu k_{\nu'}\left(1 - \frac{1}{\alpha_0^2}\right)\right] \\
&\qquad \times \left[(\mathrm{i}\omega + \gamma_0 k^2)\delta_{\mu'\sigma'} - \gamma_0 k_{\mu'} k_{\sigma'}\left(1 - \frac{1}{\alpha_0}\right)\right] \\
&\qquad \times \left[\left(\omega^2 + \gamma_0^2 \frac{(k^2)^2}{\alpha_0^2}\right)(\omega^2 + \gamma_0^2 (k^2)^2)\right]^{-1} \\
&\qquad = (-\mathrm{i}\omega + \gamma_0^2 k^2/\alpha_0)\delta_{\nu'\sigma'} \\
&\qquad\quad + \gamma_0(1 - 1/\alpha_0) k_{\nu'} k_{\sigma'}[(-\mathrm{i}\omega + \gamma_0 k^2)(-\mathrm{i}\omega + \gamma_0 k^2/\alpha_0)]^{-1} \\
&\qquad \equiv K_{\nu'\sigma'}(k, \omega).
\end{aligned} \tag{6.7}$$

Using this expression we may write

$$\begin{aligned}
(e) = 2\frac{1}{2!}\frac{g_0^2}{4} f^{abc} f^{ab'c'} &\int \frac{\mathrm{d}^d k\, \mathrm{d}\omega}{(2\pi)^{d+1}} K_{\nu'\sigma'}(k, \omega) K_{\nu\sigma}(p + k, \Omega + \omega) \\
&\times \left[\left(1 - \frac{1}{\alpha_0}\right) k_\nu \delta_{\sigma\rho} - 2k_\rho \delta_{\nu\sigma} + k_\sigma \delta_{\rho\nu}\right] \\
&\times \left[\left(1 - \frac{1}{\alpha_0}\right)(p + k)_{\nu'}\delta_{\sigma'\rho'} - 2(p + k)_{\rho'}\delta_{\sigma'\nu'} + (p + k)_{\sigma'}\delta_{\rho'\nu'}\right]
\end{aligned} \tag{6.8}$$

where the factor 2 comes from symmetry reasons as before. This expression vanishes since, in the complex ω plane, the integrand has four simple poles, and all of them have *positive* imaginary part. We could have used dimensional regularisation arguments to get rid of the above quartic divergences, but we have preferred to show that they cancel completely.

As a general rule, graphs from the Z_V rules which contain closed loops like (b) in figure 2 may safely be omitted, for any perturbative order in g_0, since they can be cancelled when added to a similar graph in which the loop has been substituted by the Euclidean volume divergence represented by (d) in figure 2. A similar cancellation among Z_V graphs which contain a closed loop (with an internal line with two crosses that starts and ends at the same point) like (a) in figure 2 and a loop like that in diagram (c) in figure 2 (where there are two circles over the same line and similarly two crosses over the other line) occurs for any perturbative order in g_0.

7. Conclusions

We have extended previous works on non-equilibrium statistical mechanics to the subject of stochastic quantisation of continuum non-Abelian YM gauge field theory. New functional integrals for the non-equilibrium and equilibrium probability distributions are given. In particular, the functional integral for the equilibrium distribution (see equation (3.4)) is manifestly non-negative for any gauge field A^a_μ and it may provide an interesting alternative with respect to the Faddeev–Popov distribution. We introduce a new dynamical generating function (4.2) related to the Fokker–Planck or, equivalently, the Langevin equations.

On the other hand, equation (4.2) is the generalisation for the YM theory of a stochastic function for critical dynamics given in [15] in which auxiliary fields, like the ones introduced by Martin *et al* [18] appear. It is possible and, we believe, advantageous, to integrate out the latter and write expression (4.2) as the starting point. This dynamical generating function is shown to reduce to the equilibrium generating function (4.1) (studied previously in [12]), in an appropriate 'static limit'. We also provide representations for the correlation functions as functional derivatives. In § 5, we give the Feynman rules associated with the generating function which include Euclidean volume divergences. As a check, the dynamical propagator is shown to reduce in the 'static limit' to the Euclidean free propagator in the Faddeev–Popov procedure.

The Feynman rules are used in § 5 to show that the superficial degree of divergence of the diagrams does not grow with the perturbative order if $d \leq 4$. In § 6, the cancellation of all Euclidean volume divergences is carried out for any dimension d for Z_V. We show that the latter cancels with some of the quartic ultraviolet divergences for $d = 4$.

We stress that it is possible, without resorting to dimensional regularisation arguments, to show the complete cancellation of all quartically divergent graphs.

In forthcoming papers, we will study the renormalisation aspects of the presented method [19].

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